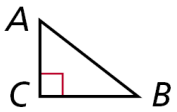
## Attendance Problems.

**1.** What are sides $AC$ and $BC$ called? Side $AB$?




**2.** Which side is between $\measuredangle A$ and $\measuredangle C$?

**3.** Given $\triangle DEF$ and $\triangle GHI$, if $\measuredangle D \cong \measuredangle G$ and $\measuredangle E \cong \measuredangle H$, why is $\measuredangle F \cong \measuredangle I$?

- I can apply ASA, AAS, and HL to construct triangles and to solve problems.
- I can prove triangles congruent by using ASA, AAS, and HL.

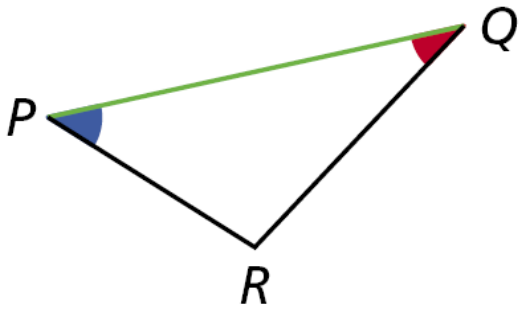
## Vocabulary: included side

### Common Core

**CC.9-12.G.CO.8** Explain how the criteria for triangle congruence (ASA, SAS, and SSS) follow from the definition of congruence in terms of rigid motions.
**CC.9-12.G.CO.7** Use the definition of congruence in terms of rigid motions to show that two triangles are congruent if and only if corresponding pairs of sides and corresponding pairs of angles are congruent.
**CC.9-12.G.SRT.5** Use congruence and similarity criteria for triangles to solve problems and prove relationships in geometric figures.

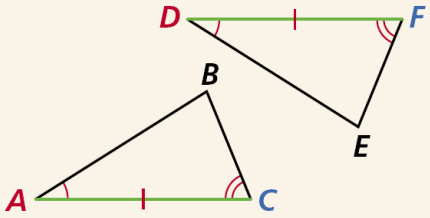
Participants in an orienteering race use a map and a compass to find their way to checkpoints along an unfamiliar course.
Directions are given by bearings, which are based on compass headings. For example, to travel along the bearing S 43° E, you face south and then turn 43° to the east.

An **included side** is the common side of two consecutive angles in a polygon. The following postulate uses the idea of an included side.




$\overline{PQ}$ is the included side of $\angle P$ and $\angle Q$.

**Postulate 4-5-1** Angle-Side-Angle (ASA) Congruence

| POSTULATE | HYPOTHESIS | CONCLUSION |
|---|---|---|
| If two angles and the included side of one triangle are congruent to two angles and the included side of another triangle, then the triangles are congruent. |  | $\triangle ABC \cong \triangle DEF$ |



**Video Example 1.** If a triangle has $m\measuredangle A = 90°, m\measuredangle B = 20°,$ and AB = 4 cm. Is the triangle unique? Explain why or why not.

SSS, SAS, and ASA are usually presented as postulates in high school texts, but in fact any one of them can be chosen as a postulate and used to prove the other two as theorems.

## 1 Problem-Solving Application

**Organizers of an orienteering race are planning a course with checkpoints *A*, *B*, and *C*. Does the table give enough information to determine the location of the checkpoints?**

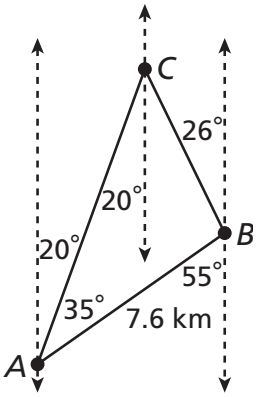
| | Bearing | Distance |
|---|---|---|
| ***A* to *B*** | N 55° E | 7.6 km |
| ***B* to *C*** | N 26° W | |
| ***C* to *A*** | S 20° W | |

### 1 Understand the Problem

The **answer** is whether the information in the table can be used to find the position of checkpoints *A*, *B*, and *C*. List the **important information:** The bearing from *A* to *B* is N 55° E. From *B* to *C* is N 26° W, and from *C* to *A* is S 20° W. The distance from *A* to *B* is 7.6 km.

### 2 Make a Plan

Draw the course using vertical lines to show north-south directions. Then use these parallel lines and the alternate interior angles to help find angle measures of $\triangle ABC$.




### 3 Solve

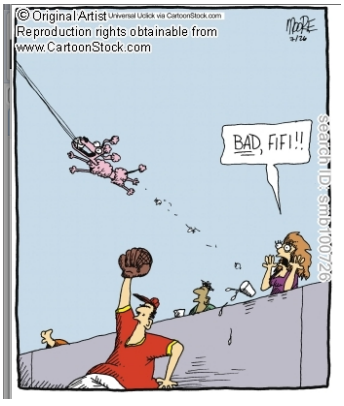
$m\angle CAB = 55° - 20° = 35°$

$m\angle CBA = 180° - (26° + 55°) = 99°$

You know the measures of $\angle CAB$ and $\angle CBA$ and the length of the included side $\overline{AB}$. Therefore by ASA, a unique triangle $ABC$ is determined.

### 4 Look Back

One and only one triangle can be made using the information in the table, so the table does give enough information to determine the location of all the checkpoints.



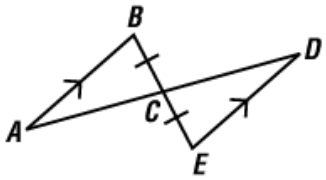
**Example 1.** A mailman has to collect mail from mailboxes at A and B and drop it off at the post office at C. Does the table give enough information to determine the location of the mailboxes and the post office?

| | Bearing | Distance |
|---|---|---|
| ***A* to *B*** | N 65° E | 8 mi |
| ***B* to *C*** | N 24° W | |
| ***C* to *A*** | S 20° W | |

**4. Guided Practice.** If 7.6 km is the distance from B to C, is there enough information to determine the location of all the checkpoints? Explain.

| | Bearing | Distance |
|---|---|---|
| ***A* to *B*** | N 65° E | 8 mi |
| ***B* to *C*** | N 24° W | |
| ***C* to *A*** | S 20° W | |

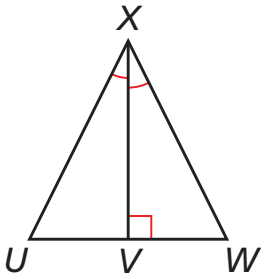
**Video Example 2.** Determine if you can use ASA to prove triangles congruent. Explain why or why not.



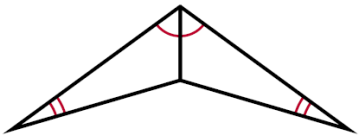

## 2 Applying ASA Congruence

**Determine if you can use ASA to prove $\triangle UVX \cong \triangle WVX$. Explain.**

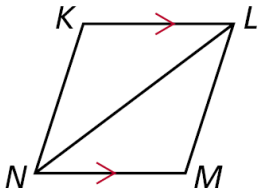
$\angle UXV \cong \angle WXV$ as given. Since $\angle WVX$ is a right angle that forms a linear pair with $\angle UVX$, $\angle WVX \cong \angle UVX$. Also $\overline{VX} \cong \overline{VX}$ by the Reflexive Property. Therefore $\triangle UVX \cong \triangle WVX$ by ASA.



**Example 2.** Determine if you can use ASA to prove the triangles congruent. Explain why or why not.

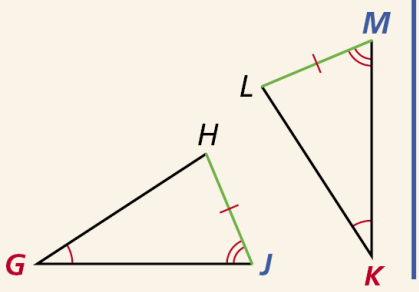


**5. Guided Practice.** Determine if you can ASA to prove $\triangle NKL \cong \triangle LMN$. Explain why or why not.




You can use the Third Angles Theorem to prove another congruence relationship based on ASA. This theorem is Angle-Angle-Side (AAS).

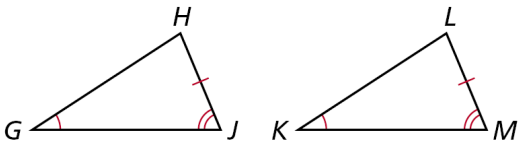
**Theorem 4-5-2** **Angle-Angle-Side (AAS) Congruence**

| THEOREM | HYPOTHESIS | CONCLUSION |
|---|---|---|
| If two angles and a nonincluded side of one triangle are congruent to the corresponding angles and nonincluded side of another triangle, then the triangles are congruent. |  | $\triangle GHJ \cong \triangle KLM$ |



**PROOF** **Angle-Angle-Side Congruence**

**Given:** $\angle G \cong \angle K, \angle J \cong \angle M, \overline{HJ} \cong \overline{LM}$
**Prove:** $\triangle GHJ \cong \triangle KLM$



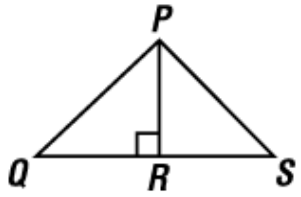

**Proof:**

| Statements | Reasons |
|---|---|
| 1. $\angle G \cong \angle K, \angle J \cong \angle M$ | 1. Given |
| 2. $\angle H \cong \angle L$ | 2. Third $\triangle$ Thm. |
| 3. $\overline{HJ} \cong \overline{LM}$ | 3. Given |
| 4. $\triangle GHJ \cong \triangle KLM$ | 4. ASA *Steps 1, 3, and 2* |

**Video Example 3.**

**Given:** $\measuredangle Q \cong \measuredangle S$
$\measuredangle QRP$ & $\measuredangle SRP$ are right angles.

Prove: $\triangle PQR \cong \triangle PSR$



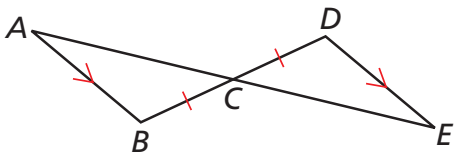

## 3 Using AAS to Prove Triangles Congruent

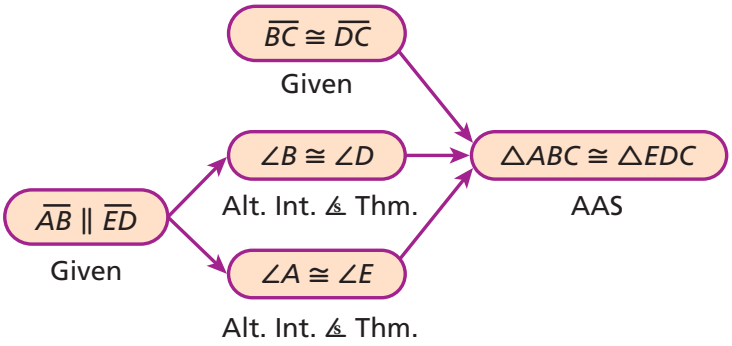
**Use AAS to prove the triangles congruent.**

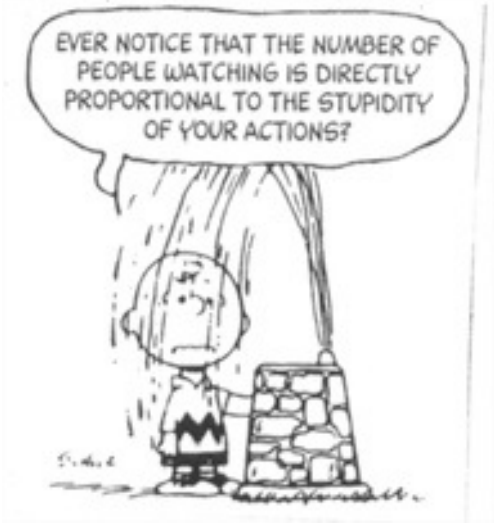
**Given:** $\overline{AB} \parallel \overline{ED}$, $\overline{BC} \cong \overline{DC}$

**Prove:** $\triangle ABC \cong \triangle EDC$

**Proof:**



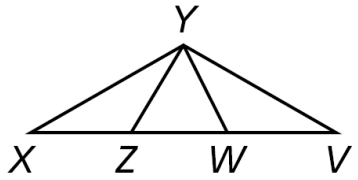
**Example 3.**

$\measuredangle X \cong \measuredangle V$

**Given:** $\measuredangle YZW \cong \measuredangle YWZ$

$\overline{XY} \cong \overline{VY}$

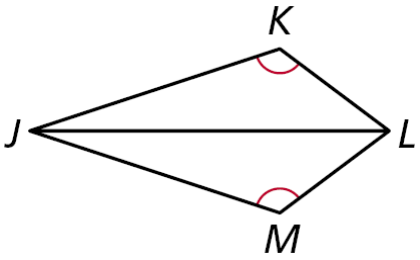
Prove: $\triangle XYZ \cong \triangle VYW$



**6. <u>Guided Practice</u>.**

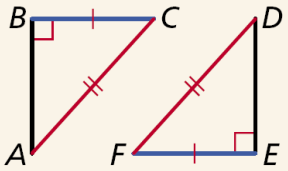
**Given:** $\overline{JL}$ bisects $\angle KLM$
$\angle K \cong \angle M$

Prove: $\triangle JKL \cong \triangle JML$




**4-6 Triangle Congruence: ASA, AAS, & HL** *(pp 265)* 9-13.

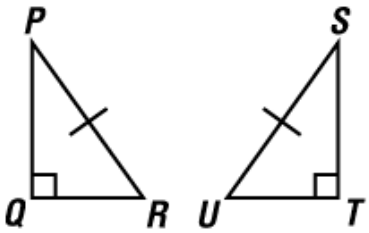
**Theorem 4-5-3** **Hypotenuse-Leg (HL) Congruence**

| THEOREM | HYPOTHESIS | CONCLUSION |
|---|---|---|
| If the hypotenuse and a leg of a right triangle are congruent to the hypotenuse and a leg of another right triangle, then the triangles are congruent. |  | $\triangle ABC \cong \triangle DEF$ |

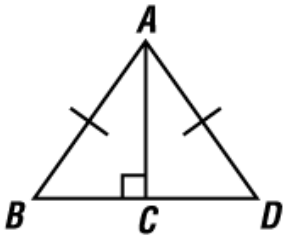


**<u>Video Example 4</u>. Determine if the two triangle are congruent. Explain why or why not.**

**A.** $\triangle PQR \cong \triangle STU$




**B.** $\triangle ACB \cong \triangle ACD$

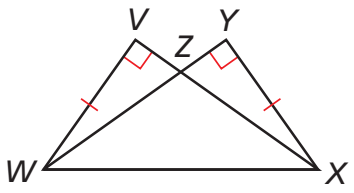


## 4 Applying HL Congruence

**Determine if you can use the HL Congruence Theorem to prove the triangles congruent. If not, tell what else you need to know.**

**A** **$\triangle VWX$ and $\triangle YXW$**

According to the diagram, $\triangle VWX$ and $\triangle YXW$ are right triangles that share hypotenuse $\overline{WX}$. $\overline{WX} \cong \overline{XW}$ by the Reflexive Property. It is given that $\overline{WV} \cong \overline{XY}$, therefore $\triangle VWX \cong \triangle YXW$ by HL.

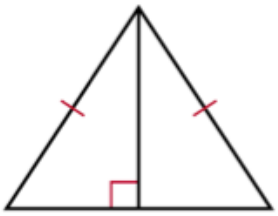



**B** **$\triangle VWZ$ and $\triangle YXZ$**

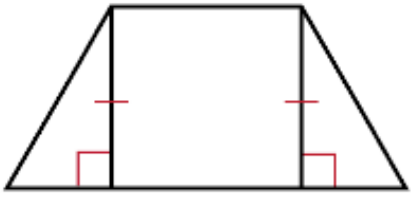
This conclusion cannot be proved by HL. According to the diagram, $\triangle VWZ$ and $\triangle YXZ$ are right triangles, and $\overline{WV} \cong \overline{XY}$. You do not know that hypotenuse $\overline{WZ}$ is congruent to hypotenuse $\overline{XZ}$.

**<u>Example 4</u>. Determine if the two triangle are congruent. Explain why or why not.**

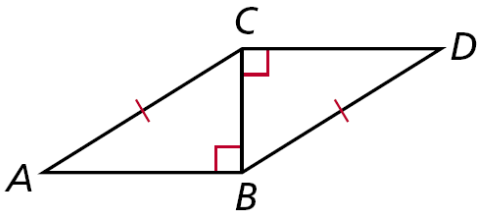
**A.**



**B.**



**7. <u>Guided Practice</u>.** Determine if $\triangle ABC \cong \triangle DCB$. Explain why or why not.




**4-6 Triangle Congruence: ASA, AAS, & HL** *(pp 265)* 9-16, 22.